Q Prove that for adiabatic expansion of ideal gas $TV^{(\gamma-1)} = \text{const.}$ $\boxed{\text{OR}}$ $PV^{\gamma} = \text{const.}$

আদর্শ গ্যাসের রুদ্ধতাপীয় প্রসারণের ক্ষেত্রে প্রমাণ কর $TV^{(\gamma-1)} =$ ধ্রুবক $\boxed{\text{বা}}$ $PV^{\gamma} =$ ধ্রুবক

Ans :- From 1st law of thermodynamics $dq = dU + W$

$$\Rightarrow 0 = dU + PdV$$

[For adiabatic process $dq = 0$
রুদ্ধতাপ প্রক্রিয়ার জন্য $dq = 0$]

$$\Rightarrow 0 = C_v dT + \frac{RT}{V}.dV$$

$\left[C_v = \left(\frac{\partial U}{\partial T}\right)_V \right.$

$\Rightarrow dU = C_v\, dT$

For 1 mole ideal gas $PV = RT$

$\Rightarrow P = \frac{RT}{V}$

$$\Rightarrow C_v dT = -\frac{RT}{V}.dV$$

$$\Rightarrow C_v \frac{dT}{T} = -R \frac{dV}{V}$$

$$\Rightarrow C_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\Rightarrow C_v \left[\ln T\right]_{T_1}^{T_2} = -R\left[\ln V\right]_{V_1}^{V_2}$$

$$\Rightarrow C_v \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow C_v \ln\left(\frac{T_2}{T_1}\right) = +(C_p - C_v) \ln\left(\frac{V_1}{V_2}\right) \qquad \{C_p - C_v = R\}$$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \frac{(C_p - C_v)}{C_v} \ln\left(\frac{V_1}{V_2}\right)$$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \left(\frac{C_p - C_v}{C_v}\right) \ln\left(\frac{V_1}{V_1}\right)$$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \left(\frac{C_p}{C_v} - \frac{C_v}{C_v}\right) \ln\left(\frac{V_1}{V_2}\right)$$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = (\gamma - 1) \ln\left(\frac{V_1}{V_2}\right)$$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{V_1}{V_2}\right)^{(\gamma-1)}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{V_1^{(\gamma-1)}}{V_2^{(\gamma-1)}}$$

$$\Rightarrow T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$

$$\Rightarrow \boxed{TV^{(\gamma-1)} = \text{Const.}}$$ Proved

$$\Rightarrow \frac{PV}{R} \cdot V^{(\gamma-1)} = \text{const.}$$

$$\Rightarrow PV^{(\cancel{1}+\gamma-\cancel{1})} = \text{Const} \times R$$

$$\Rightarrow \boxed{PV^{\gamma} = \text{Const.}}$$ Proved

$$\left[\because \frac{C_p}{C_v} = \gamma\right]$$

$$\log x^2 = 2\log x$$

For 1 mole ideal gas $PV = RT$

$$\Rightarrow T = \frac{PV}{R}$$