Work done during isothermal quasistatic process (for an ideal gas) :-

(40)

In a quasistatic isothermal expansion or compression of an ideal gas, the work is given by

$$W = -\int_{V_i}^{V_f} p\, dV ,$$

The equation of state for an ideal gas is

$$pV = nRT$$, where n and R are constants.

$$\text{or} \quad p = \frac{nRT}{V} .$$

Substituting for p in the integral, we get

$$W = -\int_{V_i}^{V_f} \frac{nRT}{V} dV .$$

Since, it is an isothermal process, T is constant.

$$\therefore \quad W = -nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$= -nRT \left[\ln V \right]_{V_i}^{V_f}$$

$$= -nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$\therefore \quad \boxed{W = -nRT \ln\left(\frac{V_f}{V_i}\right)} \longleftarrow \text{isothermal work.}$$

Derivation of adiabatic equation of state of an ideal gas from 1st law of thermodynamics

For an infinitesimal quasistatic process of a hydrostatic system, the first law is

$$đQ = dU + p\,dV$$

and the heat capacity at constant volume is given by

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V.$$

For an ideal gas U is a function of T only.

$$\therefore \left(\frac{\partial U}{\partial T}\right)_V = \frac{dU}{dT}.$$

$$\therefore C_V = \frac{dU}{dT}$$

$$\text{or } dU = C_V\,dT.$$

$$\therefore \boxed{đQ = C_V\,dT + p\,dV} \quad \text{——} \quad (1)$$

Now, all equilibrium states are represented by the ideal-gas equation

$$pV = nRT$$

For an infinitesimal process

$$p\,dV + V\,dp = nR\,dT.$$

$$\text{or } p\,dV = nR\,dT - V\,dp \quad \text{——} \quad (2)$$

from (1) & (2), we get

$$đQ = C_V\,dT + nR\,dT - V\,dp$$

$$= (C_V + nR)\,dT - V\,dp$$

or $\boxed{đQ = C_p\, dT - V\, dp}$ — (3), $C_p - C_v = nR$ for an ideal gas.

In an adiabatic process, $đQ = 0$.

$$\Rightarrow \quad C_v\, dT + p\, dV = 0$$

$$\text{or} \quad C_v\, dT = -p\, dV \quad \text{— (4)}$$

and

$$C_p\, dT = V\, dp \quad \text{— (5)}$$

$$(5) \div (4) \Rightarrow \quad \frac{C_p}{C_v} = -\frac{V\, dp}{p\, dV}$$

$$\text{or} \quad \gamma = -\frac{V\, dp}{p\, dV}$$

$$\text{or} \quad \frac{dp}{p} = -\gamma \frac{dV}{V}$$

Integrating both sides,

$$\ln p = -\gamma \ln V + \ln c$$

$$\cancel{= \ln V^{-\gamma} + \ln c} \Rightarrow \ln p + \gamma \ln V = \ln c$$

$$\text{or} \quad \ln(pV^{\gamma}) = \ln c \qquad \text{or} \quad \ln(pV^{\gamma}) = \ln c$$

$$\text{or,} \quad \boxed{pV^{\gamma} = c}$$

H.W. Problem (1) Calculate the work in compressing 2 mol of an ideal gas kept at a constant temperature of 20°C from a volume of 4 litres to 1 litres. Given: $R = 8.315$ J/mol. K.

Ans: 6753 J

Work done during a quasistatic adiabatic process (for an ideal gas):~

In a quasistatic adiabatic compression or expansion, the work is given by

$$W = -\int_{V_i}^{V_f} p\,dV .$$

In a quasistatic adiabatic process, the equation of state of an ideal gas is given by

$$pV^\gamma = c = \text{constant}, \quad \gamma = \frac{c_p}{c_v}, \text{ assumed to be constant.}$$

$$\text{or } p = \frac{c}{V^\gamma}.$$

$$\therefore W = -\int_{V_i}^{V_f} \frac{c}{V^\gamma}\,dV$$

$$= \frac{-c}{-\gamma+1}\left[V^{-\gamma+1}\right]_{V_i}^{V_f}$$

$$= \frac{c}{\gamma-1}\left[\frac{V}{V^\gamma}\right]_{V_i}^{V_f}$$

or, $W = \frac{1}{\gamma-1}\left[\frac{c}{V^\gamma}V\right]_{V_i}^{V_f}$

$= \frac{1}{\gamma-1}\left[PV\right]_{V_i}^{V_f}$

$= \frac{1}{\gamma-1}\left[P_f V_f - P_i V_i\right].$

or $\boxed{W = \frac{P_f V_f - P_i V_i}{\gamma-1}}$